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Mode Interference Effect in Coherent Electron Focusing.

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Abstract. – A novel quantum interference effect in ballistic transport is described: the interference of coherently excited magnetic edge states in a two-dimensional electron gas. The effect explains the characteristic features of the unexpected fine structure observed recently in an electron focusing experiment.

Advances in semiconductor technology have brought within reach the realization of electron optics in the solid state. In a two-dimensional electron gas (2DEG) narrow channels have been defined lithographically, through which electrons propagate with minimal scattering as in a wave guide [1, 2]. Short constrictions with a variable width of the order of the Fermi wavelength (quantum point contacts) show a quantized conductance of $2e^2/h$ per occupied subband or wave guide mode [3, 4]. The first experimental observation in a 2DEG of electron focusing by a magnetic field was reported in ref. [5], with quantum point contacts as injector and collector of ballistic electrons. At low temperatures fine structure was seen in the focusing peaks (not expected from earlier experiments in metals), which suggested that *coherent* electron focusing had been realized. This intriguing possibility is investigated theoretically in this letter. We shall demonstrate that the characteristic features of the focusing spectrum can be understood as an interference of coherently excited magnetic edge states.

The technique of electron focusing, pioneered in metals by Sharvin [6] and Tsoi [7], has become a powerful tool to investigate Fermi surfaces, boundary scattering, and the electron-phonon interaction [8]. Tsoi's transverse geometry (fig. 1) consists of two point contacts (injector and collector) on the same boundary in a perpendicular magnetic field. Electrons at the Fermi energy E_F are injected ballistically through the injector, and move in a skipping orbit along the boundary towards the collector, which serves as a voltage probe (drawing no net current). Classically, peaks in the collector voltage *vs.* magnetic-field curve

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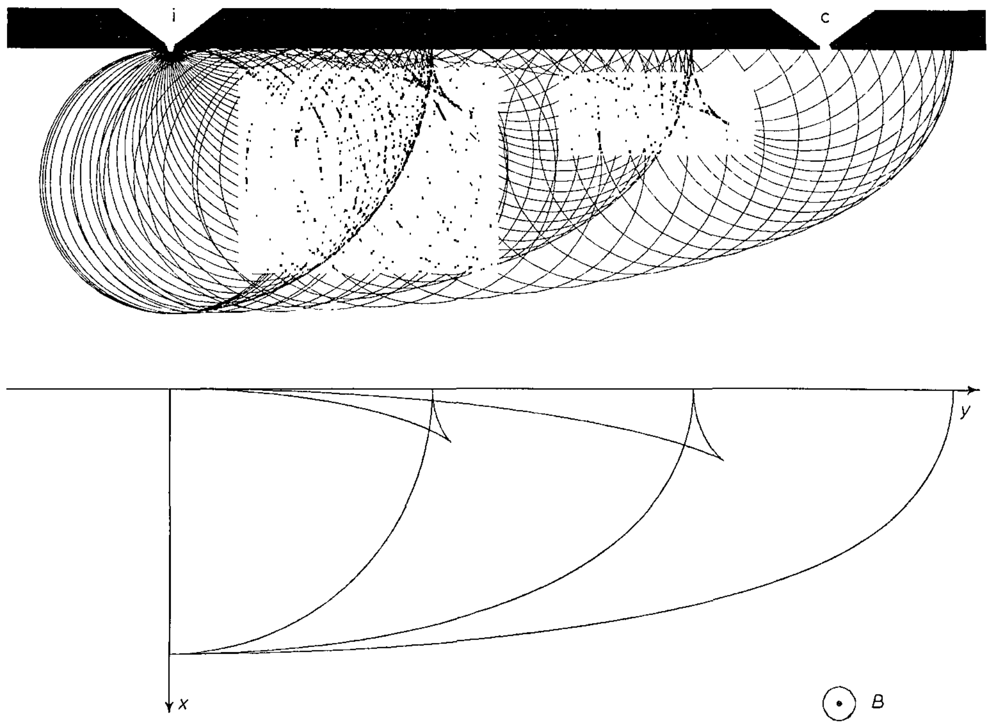

Fig. 1. – Top: Skipping orbits in a 2DEG. The gate defining the injector (i) and collector (c) point contacts and the 2DEG boundary is shown schematically in black. For clarity the trajectories are drawn only up to the third specular reflection. Bottom: calculated location of the caustic curves.

occur when a focus of the trajectories shifts past the collector. In the 2DEG of a GaAs-AlGaAs heterostructure the Fermi surface is simply a circle. The corresponding skipping orbits are shown in fig. 1. The collector coincides with a focal point when its separation L from the injector is an integer multiple of twice the cyclotron radius $l_{cycl} = \hbar k_F/eB$. Focusing thus takes place at magnetic fields B which are multiples of

$$B_{focus} = 2\hbar k_F/eL\,, \tag{1}$$

with k_F the Fermi wave vector. A simple calculation [9] of the fraction of trajectories which reach the collector from the injector predicts a series of equidistant peaks of equal height, above a monotonously increasing baseline. The p-th peak is due to electrons which have made $p-1$ specular reflections at the boundary. Such a classical focusing spectrum is commonly observed in metals, albeit with a decreasing height of subsequent peaks because of partially diffuse scattering [7-10].

In the 2DEG a strikingly different focusing spectrum is found [5]. At high temperatures (4 K) a series of focusing peaks is indeed observed at multiples of $B_{focus} = 0.066$ T, demonstrating the ballistic injection of electrons with specular reflection at the 2DEG boundary. However, upon lowering the temperature (down to 30 mK) a fine structure develops on the low-field focusing peaks, which is most pronounced as the width of the point contacts is reduced to about a Fermi wavelength ($\lambda_F \equiv 2\pi/k_F \approx 40$ nm). The fine structure grows in amplitude with the magnetic field, and at higher fields (beyond about 0.4 T) the resemblance to the classical focusing spectrum is lost. The spectrum is reproducible, but sensitive to variations in the voltage on the gate used to define the two point contacts and

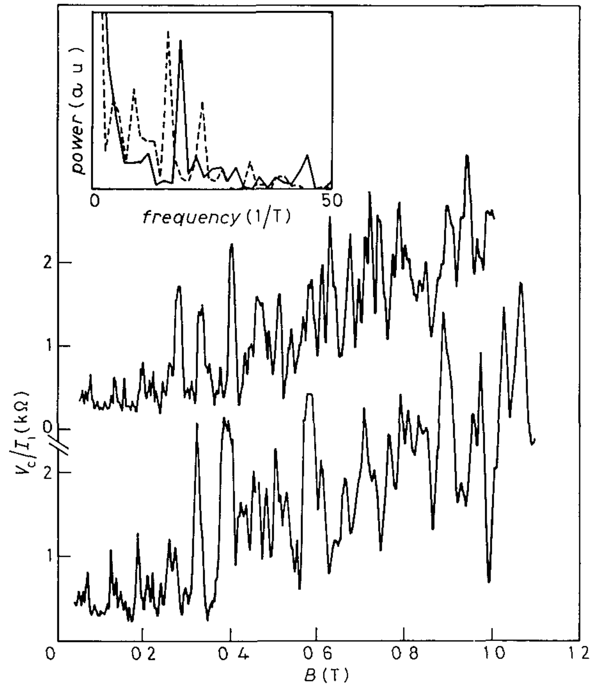

Fig. 2. – Magnetic-field dependence of the collector voltage V_c (divided by the injected current I_i), measured with the double point contact device of ref. [5] at $T = 50$ mK. Shown are the results for two gate voltages $V_g = -1.53$ V (lower trace), and $V_g = -1.22$ V (upper trace). The omitted field region below 0.05 T contains a magnetoresistance originating in the ohmic contacts, see ref. [5]. The inset gives the Fourier transform power spectrum of V_c for $B > 0.4$ T (dashed curve: $V_g = -1.53$ V, solid curve: $V_g = -1.22$ V).

the 2DEG boundary; see fig. 2, where experimental results for two different gate voltages are shown. Note that increasing the negative gate voltage has the effect of reducing the width of both point contacts. A Fourier transform of the spectra for $B > 0.4$ T (inset in fig. 2) shows that the dominant periodicity (0.06 ± 0.01) T of the high-field oscillations is approximately the same as the periodicity B_{focus} of the low-field focusing peaks. However, the amplitude is much larger.

To explain these observations it is necessary to go beyond the classical description. We first present a simple qualitative argument. Quantum ballistic transport along the 2DEG boundary takes place via magnetic edge states [11, 12], which are the propagating modes of this problem. The modes at the Fermi level are labelled by a quantum number $n = 1, 2, \ldots, n_{max}$. Since the injector has a width below λ_F, it excites these modes coherently. For $k_F L \gg 1$ the interference of modes at the collector is dominated by their rapidly varying phase factors $\exp[ik_n L]$. The wave number k_n in the y-direction (along the 2DEG) boundary, see fig. 1 for the choice of axes) corresponds classically to the x-coordinate of the centre of the cyclotron orbit, which is a conserved quantity upon specular reflection at the boundary [13]. In the gauge $A = (0, Bx, 0)$ this correspondence may be written as $k_n = k_F \sin\alpha_n$, where α is the angle with the x-axis under which the cyclotron orbit is reflected from the boundary $(-\pi/2 < \alpha < \pi/2)$. The quantized values α_n follow in this semi-classical description from the Bohr-Sommerfeld quantization rule [12, 13] that the flux enclosed by the cyclotron orbit and the boundary equals $(n - 1/4)\, h/e$ (for an infinite barrier potential). Simple geometry shows that this requires that (1)

$$\frac{\pi}{2}-\alpha_n-\frac{1}{2}\sin 2\alpha_n=\frac{2\pi}{k_F\, l_{cycl}}\left(n-\frac{1}{4}\right),\qquad n=1,2\ldots n_{max}\,, \tag{2}$$

with n_{max} the largest integer smaller than $(1/2)\,k_F\, l_{cycl}+1/4$. The dependence on n of the phase $k_n L$ is close to linear in a broad interval. This follows from expansion of eq. (2) around $\alpha_n=0$, which gives

$$k_n L=\text{constant}-2\pi n\frac{B}{B_{focus}}+k_F L\times \text{order}\left(\frac{n_{max}-2n}{n_{max}}\right)^3. \tag{3}$$

If B/B_{focus} is an integer, a fraction of order $(1/k_F L)^{1/3}$ of the n_{max} edge states interfere constructively at the collector. (The edge states outside the domain of linear n-dependence of the phase give rise to additional interference structure which, however, does not have a simple periodicity.) Because of the 1/3 power, this is a substantial fraction even for the large $k_F L\sim 450$ of the experiment. The resulting mode interference oscillations with B_{focus}-periodicity can become much larger than the classical focusing peaks. To demonstrate this, we now calculate in WKB approximation the wave function Ψ in the 2DEG.

We consider a point-dipole injector (2) and determine $|\partial\Psi/\partial x|^2$ at the coordinates $(x, y)=(0, L)$ of the collector—but unperturbed by its presence. We do not attempt to actually calculate the transmission probability from injector to collector, since this quantity is sensitive to the detailed form of the gate potential defining the point contacts and the 2DEG boundary—which we do not know. (Such a calculation would also have to take into account the reduced carrier density in the point contact region.) For point contacts with a width of the order of λ_F, the B-dependence of the collector voltage is determined in first approximation by the unperturbed probability density at an infinitesimal distance from the collector. Since for an infinite barrier potential both Ψ and $\partial\Psi/\partial y$ vanish at $x=0$, this density is proportional to our calculated $|\partial\Psi/\partial x|^2$. In the WKB approximation [14], the wave function $\Psi(x, y)$ is the sum over all classical trajectories from injector to the point (x, y) of an amplitude factor times a phase factor $\exp[i\phi]$. The amplitude factor is inversely proportional to the square root of the cross-section of a particle flux tube containing the trajectory, as required by current conservation. The phase increment ϕ acquired along the trajectory is the sum of four terms: 1) a path length term $k_F l$, with l the length of the trajectory. 2) The Aharonov-Bohm phase $(-e/\hbar)\int d\boldsymbol{l}\cdot\boldsymbol{A}$, given by the integral of the vector potential along the trajectory. In the gauge $\boldsymbol{A}=(0, Bx, 0)$ this term equals $-eBO/\hbar$, with O the area between the trajectory and the boundary at $x=0$. 3) A phase shift of π for each specular reflection at the boundary. 4) A phase shift of $-\pi/2$ for each passage through a caustic, which is a point at which the cross-section of the flux tube is reduced to zero (see fig. 1). In view of the long mean free path $l_{mfp}\sim 9\ \mu$m in the experiment [5], we do not include the effects of impurity scattering in our calculation. We have found that taking into account impurity scattering in

(1) We neglect spin-splitting, since the Zeeman energy $g\mu_B B\leqslant 10^{-3}E_F$ in the field range considered. We also neglect a possible B-dependence of k_F. In the bulk of the 2DEG, pinning of E_F at Landau levels leads to a modulation of k_F by up to 10% at 1 T. Near the boundary, however, we expect this effect to be much reduced because edge states fill the energy gap between Landau levels. Note also that since the k_F-modulation is periodic in $1/B$, it does not lead to a definite B-periodicity of the collector voltage.

(2) Dipolar injection ($\Psi\propto\cos\alpha$) was chosen instead of isotropic injection, to satisfy the boundary condition $\Psi=0$ at $x=0$.

an averaged way, by weighing the contribution of the trajectories to Ψ with a factor $\exp[-l/2l_{\mathrm{mfp}}]$, does not significantly affect our results.

The resulting magnetic-field dependence of $|\partial\Psi/\partial x|^2$ is shown in fig. 3 (bottom), for the experimental values $L = 3.0\,\mu\mathrm{m}$ and $k_{\mathrm{F}} = 1.5 \cdot 10^8\,\mathrm{m}^{-1}$ of ref. [5]. The most rapid oscillations were eliminated by averaging L over an interval of 100 nm, which corresponds roughly to

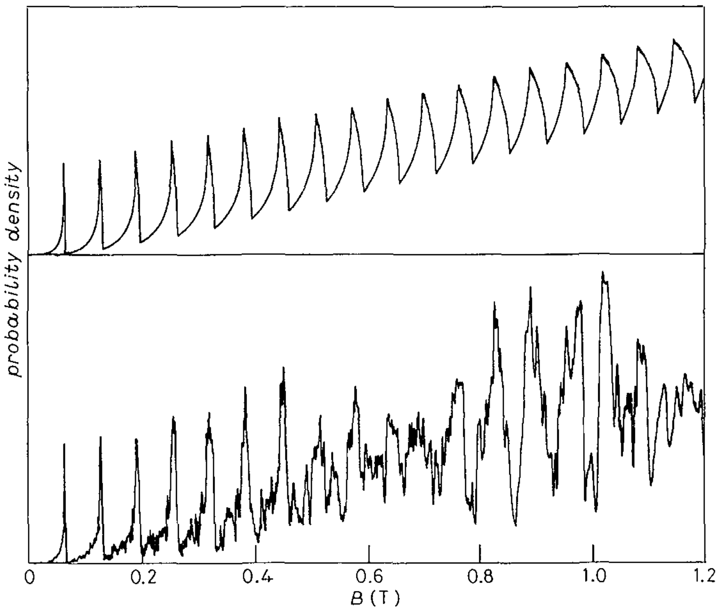

Fig. 3. – The lower curve shows the magnetic-field dependence of the unperturbed probability density at an infinitesimal distance from the collector, determined by $|\partial\Psi/\partial x|^2$. The top curve results if only the incoherent contributions are retained (no interference of skipping orbits).

the combined width of the point contacts. Also plotted in fig. 3 (top) is the incoherent contribution to $|\partial\Psi/\partial x|^2$, without the interference of different trajectories, which shows simply the peaks from classical electron focusing at multiples of B_{focus}. Interference effects give rise to fine structure on the focusing peaks at low magnetic fields, which grows in amplitude with increasing field. It is apparent from fig. 3 (and confirmed by Fourier analysis) that *the large-amplitude high-field oscillations have the same periodicity as the smaller low-field focusing peaks*—as observed experimentally, and consistent with the mode interference argument given above. This is the main result of our calculation, which we have found to be insensitive to details of the point contact modelling. (Insensitive, for example, to assuming isotropic instead of dipolar injection.) The relation between the above description in terms of interfering skipping orbits and the description in terms of interfering edge states used in the qualitative argument can be made explicit, by transforming the sum over trajectories into a sum over modes using the method of stationary phase. We have verified in this way that the phases of the modes are indeed determined by eq. (2). These two alternative representations of the quantum-mechanical transport problem are the analogues of the classical ray and mode descriptions of propagation in a wave guide. In this context the edge states correspond to Lord Rayleigh's «whispering gallery» waves [15].

We note that Tsoi [16] (to explain a fine structure in the first focusing peak in bismuth) has proposed that an *individual* edge state n would cause a peak in the collector voltage whenever L is an integer multiple of the chord $2l_{\mathrm{cycl}}\cos\alpha_n$ of the corresponding skipping orbit. We do not see how this can be reconciled with the fact that the probability density $|\Psi_n|^2$ of an individual edge state is y-independent (since $\Psi_n(x, y) = f_n(x) \exp[ik_n y]$).

A quantitative comparison between theory and experiment requires a more detailed modelling of the point contacts and gate potential. The appearance of high-field oscillations with the focusing periodicity but with much larger amplitude is, however, characteristic of the mode interference mechanism proposed in this letter (and is indeed the feature which experimentally is insensitive to small changes in gate voltage). This novel quantum interference effect in ballistic transport described here for the double point contact geometry of ref. [5] may also play a role in the multi-probe «electron wave guides» [1] of current interest. Voltage fluctuations with a well-defined periodicity were found in such a device by Chang *et al.* [1], albeit in the regime where the transport was not fully ballistic. Our demonstration of coherent electron focusing shows that interference experiments can be realized using quantum point contacts as monochromatic electron sources. These may be seen as the first proven building blocks of electron optics in the solid state.

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